Practice problems on spherical trigonometry.

**Problem 1.** Find the missing sides and angles in each of the following cases for a spherical triangle $ABC$:
(a) $a = 60^\circ$, $\beta = 90^\circ$, $\gamma = 75^\circ$.
(b) $\alpha = 65$, $\beta = 85$, $\gamma = 90$.
(c) $a = 90$, $b = 60$, $c = 100$.
(d) $\alpha = 85$, $b = 95$, $c = 105$.

**Problem 2.** In a spherical triangle $ABC$ do the following properties hold?
(a) If $AB = AC$ are the base angles at $B$ and $C$ equal?
(b) If the angles at $B$ and $C$ are equal is it true that $AB = AC$?
(c) Do the angles add to $180^\circ$?
(d) Do the sides add to $180^\circ$?
(e) If $C = 90^\circ$ is it true that $AB^2 = BC^2 + CA^2$?
(f) Do two triangles with equal corresponding sides have equal corresponding angles?
(g) Do two triangles with equal corresponding angles have equal corresponding sides?

**Problem 3.** Suppose that $P$ is the north pole and points $X$ and $Y$ in the northern hemisphere are $45^\circ$ apart and form a triangle $PXY$ with angles $60^\circ$ at $X$ and $80^\circ$ at $P$. Find the latitude of $Y$. Can you determine the longitude of $Y$?

**Problem 4.** Two points on the earth have latitude and longitude coordinates as follows: $A = (45^\circ N, 60^\circ W)$, $B = (60^\circ N, 0^\circ W)$. What direction should a plane fly to follow a great circle route from $A$ to $B$? (Give your answer as the angle made to the direction of north at $A$.)

**Problem 5.** In a spherical triangle the angles at $\alpha$, $\beta$ and $\gamma$ are $\pi/5$, $\pi/3$, $\pi/2$. Find the sum of the sides.

**Problem 6.** In a right angled spherical triangle $\alpha = a \neq 90^\circ$. Find $b$ and $c$.

**Problem 7.** In an equilateral spherical triangle show that $\sec\alpha = 1 + \sec\alpha$.

**Problem 8.** Suppose that $A$, $B$, $C$ and $X$ are four points on the surface of a sphere. Such that:
(i) The point $X$ lies on the geodesic from $B$ to $C$.
(ii) The angles at $A$, $B$, and $X$ of the spherical triangle $ABX$ are $60^\circ$, $60^\circ$, and $90^\circ$.
(iii) The geodesics $AB$ and $AC$ make an angle of $90^\circ$.
Find the measures (in either degrees or radians) of the geodesics $AB$, $AX$, $BX$, $CX$, and $AC$ and find the area of the spherical triangle $ABC$.

**Answer:**

**Problem 9.** Suppose that $A$, $B$, $C$ and $X$ are four points on the surface of a sphere. Such that:
(i) The point $X$ lies on the geodesic from $B$ to $C$ (between $B$ and $C$).
(ii) The angles at $A$, $B$, and $X$ of the spherical triangle $ABX$ are $60^\circ$, $45^\circ$, and $90^\circ$.
(iii) The geodesics $AB$ and $AC$ make an angle of $90^\circ$.
Find the measures (in either degrees or radians) of the geodesics $AB$, $AX$, $BX$, $CX$, and $AC$.

**Answer:**