Practice problems on affine geometry.

Problem 1 Find an affine transformation that turns the points $(1,0)$, $(3,5)$, $(2,3)$ into the points $(0,1)$, $(3,2)$, $(4,5)$, respectively. (Give a formula $f(\underline{x}) = M\underline{x} + \underline{b}$ for the transformation.)

Problem 2 Let $ABCD$ be a parallelogram. Let X and Y be trisection points of the diagonal AC (where X is closer to A and Y is closer to C). Let $R = BX \cap AD$ and $S = BY \cap CD$. Let f be an affine transformation that maps B to $B' = (0,0)$, A to $A' = (3,0)$, and C to $C' = (0,3)$.

(a) Find the co-ordinates of the images D', X', Y', R' and S' of D, X, Y, R, and S under f. Give brief reasons for your answers.

(b) Find the slopes of $A'C'$ and $R'S'$.

(c) What can you say about the type of the quadrilateral $ACSR$? (Is it a square, rectangle, parallelogram, $\ldots$).

Problem 3 Prove the following theorem. (Hint: Use an affine transformation to an easier configuration.)

If a trapezoid is inscribed in an ellipse, then the line joining the midpoints of the two parallel sides of the trapezoid passes through the center of the ellipse.

Problem 4 Which of the following geometrical configurations are affine concepts? Answer with yes for any affine concepts and no for non-concepts.

(a) parallelogram

(b) trapezoid

(c) rectangle

(d) isosceles trapezoid

(e) conic

(f) ellipse

(g) ratio of perpendicular segments

(h) ratio of parallel segments

(i) incircle of a triangle

(j) collinear triple of points, one on each of three sides of a triangle

Problem 5 Let ABC be a triangle. Let BD be a median of triangle ABC. Let X and Y be two points on the segment BC such that $BX = XY = YC$. (So X and Y divide BC into 3 equal parts.) Let AX meet BD at Z. Use affine geometry to prove that Z is the midpoint of BD.

Problem 6 Let $ABCD$ be a parallelogram. Let X and Y be the midpoints of the sides AB and BC. Let AY meet CX at Z. Use affine geometry to prove that Z lies on the diagonal BD and to determine the ratio DZ/ZB.

Problem 7 Let ABC be a triangle. Let BD be a median of triangle ABC. Let X, Y, and Z be three points on the segment BC such that $BX = XY = YZ = ZC$. (So X, Y and Z divide BC into 4 equal parts.) Let AX meet BD at P. Use affine geometry to prove that $AP/AX = 4/5$.

Problem 8 Let ABC be a triangle and let D be a point on the side BC. Let X and Y be the centroids of triangles ABD and ACD. Use affine geometry to prove that the line XY is parallel to BC.

Problem 9 Let T be a trapezoid. Prove that there is an affine transformation that transforms T to an isosceles trapezoid. (An isosceles trapezoid is a trapezoid with two equal base angles.)